

# Cambridge O Level

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**ADDITIONAL MATHEMATICS****4037/12**

Paper 1 Non-calculator

**May/June 2025**

MARK SCHEME

Maximum Mark: 80

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Published

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This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

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This document consists of **13** printed pages.

**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

**GENERIC MARKING PRINCIPLE 1:**

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

**GENERIC MARKING PRINCIPLE 2:**

Marks awarded are always **whole marks** (not half marks, or other fractions).

**GENERIC MARKING PRINCIPLE 3:**

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

**GENERIC MARKING PRINCIPLE 4:**

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

**GENERIC MARKING PRINCIPLE 5:**

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

**GENERIC MARKING PRINCIPLE 6:**

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

**Annotations**

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Accuracy mark awarded two
	Accuracy mark awarded three
	Independent mark awarded zero
	Independent mark awarded one
	Independent mark awarded two
	Independent mark awarded three
	Benefit of the doubt
	Communication mark
	Incorrect
	Follow through
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Method mark awarded three

Annotation	Meaning
<b>MR</b>	Misread
<b>O</b>	Omission
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
<b>Pre</b>	Premature rounding/approximation
<b>SC</b>	Special case
<b>SEEN</b>	Indicates that work/page has been seen
<b>TE</b>	Transcription error
	Correct
<b>XP</b>	Correct answer from incorrect working

**MARK SCHEME NOTES**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

**Types of mark**

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘dep’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

**Abbreviations**

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	$\pm \frac{1}{3}(x+2)(2x+1)(x-3)$ isw	3	<p><b>B1</b> for <math>\pm</math> with 3 factors or expanded out cubic</p> <p><b>B1</b> for <math>\frac{1}{3}</math> in a product with 3 factors</p> <p><b>B1</b> for <math>(x+2)(2x+1)(x-3)</math> and no extra terms or <math>2(x+2)(x+0.5)(x-3)</math> and no extra terms</p>
2	$x^{\frac{2}{3}} + x^{\frac{1}{3}} - 6 (=0)$ oe soi	B1	<p>For multiplying through by <math>x^{\frac{1}{3}}</math></p> <p>Allow if substitution is used</p>
	$\left(x^{\frac{1}{3}} + 3\right)\left(x^{\frac{1}{3}} - 2\right) (=0)$ oe	M1	Allow if substitution is used
	$x = -27$	A1	<b>A0</b> if rejects $x = -27$
	$x = 8$	A1	
3(a)	$5x^2 - 30x + 45 (=0)$ or equivalent 3 term quadratic	2	<b>M1</b> for attempt to eliminate one variable
	Use of discriminant or solution to show one repeated root or a single solution	A1	Must have conclusion e.g. Discriminant = 0 (so tangent) oe One solution only (so tangent)
	<b>Alternative 1</b>		
	Centre of the circle $(5, 2)$ Equation of radius perpendicular to given line $y - 2 = -\frac{1}{2}(x - 5)$ Intersection at $(3, 3)$	(2)	<b>M1</b> for a complete method to obtain the point of intersection
	Show that P lies on the circle or that $CP = \sqrt{5}$	(A1)	
	<b>Alternative 2</b>		
	The line $y = kx + n$ touches the circle $(x - p)^2 + (y - q)^2 = r^2$ if $(-kp + q - n)^2 = r^2(1 + k^2)$ so $((-2 \times 5) + 2 - 3)^2 = 5(1 + 2^2)$	(2)	<b>M1</b> for a complete method showing substitution, allow one slip
	$25 = 25$ , so a tangent	(A1)	

Question	Answer	Marks	Guidance
3(b)	(3, 3)	2	B1 for each
3(c)	Radius = $3\sqrt{2}$ oe soi	B1	
	$(x-3)^2 + (y-3)^2$ soi	B1	FT on <i>their</i> answer to (b) only
	$(x-3)^2 + (y-3)^2 = 18$ oe	B1	ISW any expansion
4(a)	$[-\cos \theta]_0^\pi$	B1	May be implied by $(-\cos \pi) - (-\cos 0)$
	2	B1	
4(b)	$\frac{1}{\cos \alpha} \quad \frac{\cos \alpha + \sin \alpha}{\sin \alpha \cos \alpha}$	B1	For ratios in terms of sine and cosine
	$\frac{1}{\cos \alpha} \quad \frac{\cos^2 \alpha + \sin^2 \alpha}{\sin \alpha \cos \alpha}$	B1	For simplifying the denominator to one term
	$\frac{1}{\cos \alpha} \quad \frac{1}{\cos \alpha \sin \alpha}$ = $\sin \alpha$	B1	Must show sufficient correct detail
	<b>Alternative</b>		
	$\frac{1}{\cos \alpha} \quad \frac{\sec \alpha}{\tan \alpha + \tan \alpha}$ or $\frac{1}{\tan \alpha} + \tan \alpha$	(B1)	For ratios in terms of tangent and cosine or secant and tangent
	$\frac{1}{\cos \alpha} \quad \frac{\sec \alpha}{\sec^2 \alpha}$ or $\frac{\sec \alpha}{\tan \alpha}$ oe	(B1)	For dealing with the denominator
	$\frac{\tan \alpha}{\sec \alpha} = \frac{\sin \alpha}{\cos \alpha} \times \cos \alpha$ oe = $\sin \alpha$	(B1)	Must show correct sufficient detail

Question	Answer	Marks	Guidance
5(a)	$p'(x) = 9x^2 - 14x + a$ soi	<b>B1</b>	
	$p'(-1) = 9 + 14 + a = 21$ $a = -2$	<b>B1</b>	
	$p(2) = 24 - 28 + 2a + b = 0$ soi	<b>B1</b>	Each term must be simplified, allow using <i>their a</i>
	$b = 8$	<b>B1</b>	
5(b)	$[x-2](3x^2 - x - 4)$ soi	<b>2</b>	<b>M1</b> for quadratic factor with two terms correct <b>A1</b> must be from correct $a$ and $b$
	$(x-2)(x+1)(3x-4)$	<b>A1</b>	<b>A1</b> must be from correct $a$ and $b$
5(c)	$(e^{2y} - 2) = 0$ soi	<b>B1</b>	
	$y = \frac{1}{2} \ln \frac{4}{3}$ oe, $y = \frac{1}{2} \ln 2$ oe	<b>2</b>	<b>B1</b> for one correct solution <b>B2</b> for both solutions and no extra solution.
6(a)	$\ln y = mx^3 + c$ soi	<b>B1</b>	$c$ may be shown as a log term
	$25 = -8m + c$ $5 = 2m + c$	<b>M1</b>	For at least one correct equation, may be used with gradient of $-2$
	$m = -2, c = 9$	<b>A1</b>	$y = -2x + 9$ seen implies <b>M1A1</b>
	$y = e^{9-2x^3}$ oe	<b>A1</b>	
6(b)	$25 = 9 - 2x^3$	<b>M1</b>	For equating exponential indices and obtaining $x = \dots$ or use of $x = \sqrt[3]{\frac{25 - \text{their } 9}{\text{their } - 2}}$
	$x = -2$ only	<b>A1</b>	

Question	Answer	Marks	Guidance
7(a)	$ar^3 = \frac{8k^6}{27}, ar^5 = \frac{32k^{10}}{243}$ soi	B1	May be implied by a correct 5th term $\frac{16k^8}{81}$
	$r = \frac{2k^2}{3}$	2	<b>M1</b> for solution of <i>their</i> equations to obtain either $r = mk^2$ or $a = c$ , where $m$ is an unsimplified numeric constant and $c$ is a non-zero constant  e.g. Use of $r^2 = \frac{6\text{th term}}{4\text{th term}}$
	$a = 1$	A1	
7(b)	$\frac{1}{1 - \frac{2k^2}{3}} = 3$ soi	B1	For use of sum to infinity formula with <i>their</i> values of $a$ where $a$ is numeric and $r$ which is in terms of $k^2$
	$k = \pm 1$	2	<b>B1</b> for each must be from correct work
8(a)	$\left( \frac{dy}{dx} = \right) \frac{(x+2) \frac{6x}{3x^2+16} - \ln(3x^2+16)}{(x+2)^2}$	3	<b>B1</b> for $\frac{6x}{3x^2+16}$ <b>M1</b> for differentiation of a quotient or equivalent product <b>A1</b> for all other terms apart from $\frac{6x}{3x^2+16}$ correct
	When $x = 0$ , $\frac{dy}{dx} = -\frac{\ln 16}{4}$ $= \ln \frac{1}{2}$ from correct work	2	<b>M1 dep</b> for attempt to substitute in $x = 0$ and obtain a single log term For <b>A1</b> , all previous marks must have been awarded.
8(b)	Change = $h \ln \frac{1}{2}$ oe	B1	<b>FT</b> on <i>their</i> single log answer to part (a) <b>B0</b> for $\ln \frac{1}{2}h$ or <i>their</i> $\ln \frac{1}{2}h$
9(a)	$\frac{4}{3}$ or $a = \frac{4}{3}$	B1	Award <b>B1</b> when a correct answer is seen
9(b)	$\mathbb{R}$ oe	B1	May be in terms of $f$ or $y$ but not $x$
9(c)	$[f^{-1}(x)] = \frac{4 + e^{\frac{x}{2}}}{3}$ oe	2	<b>M1</b> for a complete valid method, allow a sign error <b>A1</b> must be using the correct notation

Question	Answer	Marks	Guidance
9(d)		4	<p><b>B1</b> for <math>y = f(x)</math> in the 1<sup>st</sup> and 4<sup>th</sup> quadrants and appropriate asymptotic behaviour</p> <p><b>B1 dep</b> on previous B mark for <math>y = f^{-1}(x)</math> intersecting twice with <math>y = f(x)</math>, in the 1<sup>st</sup> and 2<sup>nd</sup> quadrants and appropriate asymptotic behaviour</p> <p><b>B1 dep</b> on first <b>B1</b> for <math>\frac{5}{3}</math> marked correctly on each axis or stated, and no other intercepts</p> <p><b>B1</b> for <math>y = \frac{4}{3}</math> and <math>x = \frac{4}{3}</math> (independent) either on the graph or at the side.</p>
10(a)	$\frac{1}{2} \left( \frac{4r}{3} \right)^2 \theta - \frac{1}{2} r^2 \sin \theta$ oe	3	<p><b>B1</b> for <math>\frac{1}{2} \left( \frac{4r}{3} \right)^2 \theta</math></p> <p><b>B1</b> for <math>\frac{1}{2} r^2 \sin \theta</math> or <math>r^2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}</math> or <math>\frac{r^2 \sin^2 \frac{\theta}{2}}{\tan \frac{\theta}{2}}</math> oe</p> <p>Allow final answer unsimplified</p>
10(b)	$\text{Arc length} = r \frac{(\pi - \theta)}{2}$	<b>B1</b>	
	$BE = \sqrt{2r^2(1 - \cos \theta)}$ or $2r \sin \frac{\theta}{2}$ or $\frac{r \sin \theta}{\sin \left( \frac{\pi - \theta}{2} \right)}$ oe	2	<p><b>M1</b> for complete method to find <math>BE</math> or <math>BE^2</math>, using either cosine rule, sine rule or basic trig using <math>r</math> and a correct angle.</p> <p><b>A0</b> if an error in rearranging to obtain <math>BE</math> is made.</p> <p>Allow unsimplified</p>
	$\text{Perimeter} = r(\pi - \theta) + 2r + \sqrt{2r^2(1 - \cos \theta)}$ oe	2	<p><b>M1</b> for a correct plan using <i>their</i> lengths</p> <p><math>2 \text{ arc lengths} + 2r + \text{their } BE</math></p> <p>Allow <b>A1</b> for an unsimplified answer</p>

Question	Answer	Marks	Guidance
11(a)	$\overrightarrow{OQ} = \mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a})$ or $\overrightarrow{OQ} = \mathbf{b} + \frac{2}{3}(\mathbf{a} - \mathbf{b})$ $\left( \overrightarrow{OQ} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} \right)$	2	<b>M1</b> for attempt to use $\overrightarrow{OQ} = \mathbf{a} + k(\mathbf{b} - \mathbf{a})$ or $\overrightarrow{OQ} = \mathbf{b} + k(\mathbf{a} - \mathbf{b})$ , where $k$ can be positive or negative Allow unsimplified
	$\overrightarrow{PQ} = -\frac{3}{4}\mathbf{a} + \text{their } \overrightarrow{OQ}$	<b>M1</b>	
	$\overrightarrow{PQ} = -\frac{3}{4}\mathbf{a} + \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ $\left( \overrightarrow{PQ} = \frac{1}{3}\mathbf{b} - \frac{1}{12}\mathbf{a} \right)$	<b>A1</b>	Allow unsimplified
	$\overrightarrow{OP} = \frac{3}{4}\mathbf{a}$	<b>B1</b>	Allow anywhere
	$\overrightarrow{OR} = \frac{3}{4}\mathbf{a} + \mu\left(\frac{1}{3}\mathbf{b} - \frac{1}{12}\mathbf{a}\right)$ oe	<b>B1</b>	Must be simplified
<b>Alternative</b>			
	$\overrightarrow{PQ} = \frac{1}{4}\mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a})$ $\left( \overrightarrow{PQ} = \frac{1}{3}\mathbf{b} - \frac{1}{12}\mathbf{a} \right)$	(4)	<b>B1</b> for $\overrightarrow{PA} = \frac{1}{4}\mathbf{a}$ <b>M1</b> for $\overrightarrow{AQ} = \frac{1}{3}(\mathbf{b} - \mathbf{a})$ , allow a sign error i.e. use of $(\mathbf{a} - \mathbf{b})$ <b>M1</b> for attempt to use $\overrightarrow{PQ} = \text{their } \frac{1}{4}\mathbf{a} + \text{their } \overrightarrow{AQ}$ Allow unsimplified
	$\overrightarrow{OP} = \frac{3}{4}\mathbf{a}$	(B1)	
	$\overrightarrow{OR} = \frac{3}{4}\mathbf{a} + \mu\left(\frac{1}{3}\mathbf{b} - \frac{1}{12}\mathbf{a}\right)$ or $\overrightarrow{OR} = \frac{3}{4}\mathbf{a} - \frac{\mu}{12}\mathbf{a} + \mu\left(\frac{1}{3}\mathbf{b}\right)$ or $\overrightarrow{OR} = \left(\frac{3}{4} - \frac{\mu}{12}\right)\mathbf{a} + \frac{\mu}{3}\mathbf{b}$	(B1)	Must be simplified

Question	Answer	Marks	Guidance
11(b)	$\lambda = \frac{1}{3}\mu$	<b>B1</b>	For equating <b>b</b> vectors
	$\frac{3}{4} - \frac{1}{12}\mu = 0$ oe	<b>B1</b>	For equating <b>a</b> vectors to zero
	$\mu = 9, \lambda = 3$	<b>B1</b>	
12	When $\frac{dy}{dx} = 0, x = \frac{2}{5}$	<b>B1</b>	Must come from first derivative and not the second derivative
	$(y =) \frac{3}{20}(5x-2)^{\frac{4}{3}} (+c)$	<b>2</b>	<b>M1</b> for $k(5x-2)^{\frac{4}{3}}$
	$\frac{32}{5} = \frac{3}{20}((5 \times 2) - 2)^{\frac{4}{3}} + c$ oe	<b>M1</b>	<b>Dep</b> for attempt to find $c$ using <i>their</i> $k(5x-2)^{\frac{4}{3}}$
	$c = 4$	<b>A1</b>	Allow an unsimplified fraction
	Stationary point $\left(\frac{2}{5}, 4\right)$	<b>B1</b>	<b>Dep</b> on first <b>B</b> mark